An Analytical Non-Dimensional Model for Mode I Delamination of Z-Pinned Composites

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ABSTRACT

This paper presents an analytical non-dimensional model to analyze crack propagation in a z-pinned double cantilever beam specimen (DCB) under Mode I loading. Effect of various design parameters on the crack bridging length and apparent fracture toughness are investigated using this model. The efficacy of the analytical model is evaluated by comparing the same with 3D FE simulations of the DCB. In the FE model the z-pins are modeled as discrete non-linear elements. Bi-linear cohesive elements are used ahead of the crack tip to account for the inherent fracture toughness of the composite material. The results for load-deflection and crack length obtained from the analytical model and the FE model are compared and found to be in good agreement. Proposed non-dimensional analytical model will be useful in the design of translaminar reinforcements for composite structures.

INTRODUCTION

Delamination is one of the most significant failure modes in laminated composites. Translaminar reinforcements can enhance the delamination resistance and damage tolerance of composites significantly [1]. Stitching, z-pinning and 3D weaving are some of the methods by which translaminar reinforcements can be provided in a composite laminate. It is also known that sparse through-the-thickness reinforcements are not beneficial, and at the same time excessive amount of reinforcement also results in damage to the composite structure further degrading its properties [2].

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The effect of z-fiber on delamination of composites has been studied extensively. While FE models that account for each and every reinforcement may be more accurate in predicting the damage tolerance, they tend to be computationally expensive for realistic structures. On the other hand analytical models that smear the reinforcements as a continuous element are efficient and provide insight into the mechanics, but tend to be unrealistic in some situations.

Experimental approaches involve measurement of apparent fracture toughness [3], characterization of pull-out process of z-fiber [4] and crack propagation [5]. Analytic models are also available to find apparent fracture toughness and to establish the relationship between various design parameters [6-8]. Numerical studies focused on crack initiation and progressive delamination modeling in which J-integral, VCCT or cohesive zone method [9-11] were primarily used.

In this paper the effect of z-pins on the fracture toughness of laminated composites is investigated. A non-dimensional analytical model is proposed. Bridging zone developed by the z-pins is analyzed using a beam on elastic foundation model although the foundation is neither elastic nor linear. The model is used to find the effect of various parameters of the z-pinned DCB specimen on the force-deflection relation and the force-crack length relation. The analytical model is non-dimensionalized so that it can serve as a design tool in selecting appropriate translaminar reinforcements, z-pins in this case. An expression for apparent fracture toughness or effective fracture toughness is derived. Furthermore, the maximum pin friction that can be allowed before the composite beam itself fails is calculated.

The efficacy of the analytical model is verified by finite element simulation of the DCB specimen. In the FE simulation, the ligaments of the DCB are modeled using shell elements. Cohesive elements are used to simulate the delamination and discrete non-linear elements are used to model the z-pins. The agreement between the analytical model and FE simulations is found to be excellent for various results such as load-deflection, load-crack length and effective fracture toughness. The non-dimensional analytical model could be a useful design tool in selecting z-pins for composite structures to improve interlaminar fracture toughness

ANALYTIC MODEL

The DCB specimen analyzed is shown in Fig. 1. The initial crack length (AB in Fig.1) is a_0. The current crack length (AD) is denoted as a. The region between the initial crack tip and the current crack tip (BD), which is of length of (a-a_0), consists of a bridging zone CD of length c and a zone BC of length (a_p-a_0) where the pins have been completely pulled out of the composite beam. The distance a_p (AC in Fig.1) can be thought of as the apparent crack length. In the bridging zone the pull-out of pins is partial. Beyond the current crack tip D the pins are assumed to be intact. A pair of transverse forces F is applied at the tip of the DCB. The relative deflection of the DCB ends is δ. Our goal is to determine the relationships among F, a, c and δ.

We assume the shear deformation is negligible and use Euler-Bernoulli beam equations to model the ligaments of the DCB. We assume that there is sufficient friction between the z-pins and the composite material surrounding it. The relation between the frictional force (f) and pull out or slip distance (d_s) is idealized as shown in Fig. 2. When the pins are intact they can exert a maximum friction force of f_m. As the pins pull out of the material, the loss of friction is proportional to the pullout distance d_s. When the pin is completely out of the beam, the friction force reduces to zero. Thus the f-d_s relationship is given by

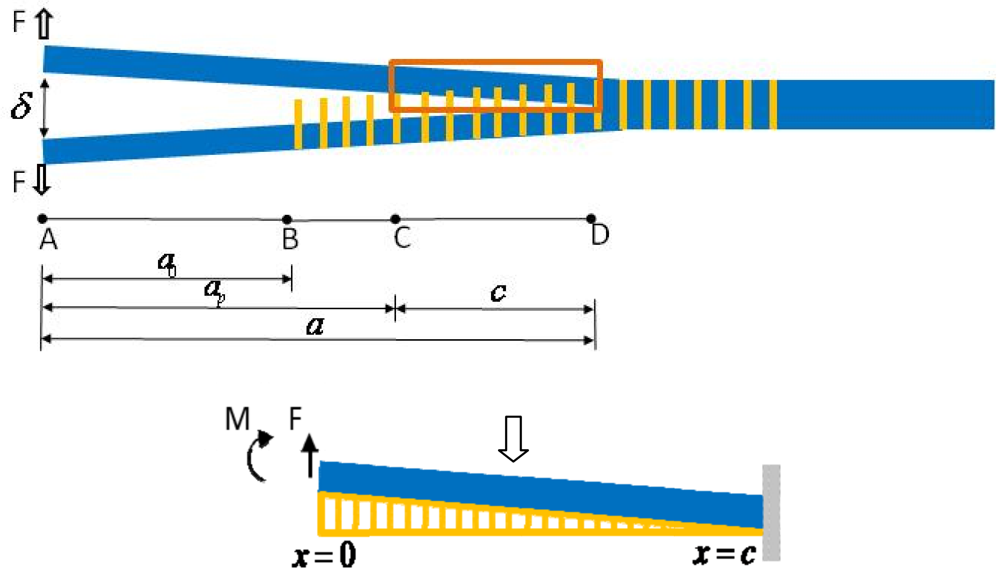

Figure 1: Various dimensions of the DCB and bridging zone developed by the z-pins

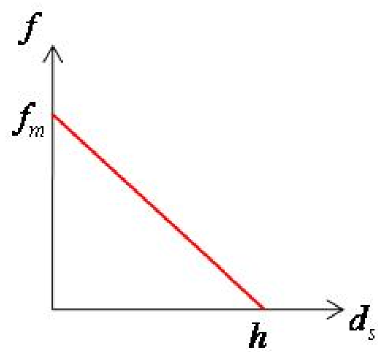

Figure 2: Force-displacement relation of the z-pin

$$f = f_m\left(1-\frac{d_s}{h}\right), 0 \le d_s \le h \tag{1}$$

Although the force exerted by the pins on the beam is discreet, for the purpose of the analytical model we smear the discrete pin forces as continuous distributed traction acting on the crack surfaces. Then the traction can be derived as

$$p = Nf = Nf_m\left(1-\frac{d_s}{h}\right) = p_m\left(1-\frac{d_s}{h}\right) \tag{2}$$

where N is z-pin density expressed as number of z-pins/unit area.

The Euler-Bernoulli beam equation for one of the ligaments, say upper ligament, of the DCB can be written as:

$$EI\frac{d^4w}{dx^4} = -bp \tag{3}$$

where b is the beam width in the y-direction. The effective bending rigidity of one of the ligaments of the DCB is represented by the term EI. For a laminated composite the flexural rigidity can be taken as $EI=bD_{11}$. Substituting for p from Eq. (3), we obtain

$$EI\frac{d^4w}{dx^4} - bp_m\frac{d_s}{h} = -bp_m \tag{4}$$

The pullout length d_s is actually equal to $2w$, where w is the deflection of the top or bottom beam. Hence, the governing equation takes the form

$$EI\frac{d^4w}{dx^4} - 2p_m\frac{w}{h} = -bp_{\mathrm{m}} \tag{5}$$

The origin of the x-coordinate is assumed to be at C, beginning of bridging zone. One should note that the origin moves as the crack propagates. The four boundary conditions are:

$$\begin{aligned}
&w(0) = w_0, \left(0 < w_0 \le h/2\right)\\
&M_x(0) = a_p V_z(0) \Rightarrow \frac{d^2w}{dx^2}(0) = a_p\frac{d^3w}{dx^3}(0)\\
&w(c) = 0\\
&\frac{dw}{dx}(c) = 0
\end{aligned} \tag{6}$$

In the above equation w_0 is a prescribed deflection at Point C in Fig. 1. This value will be less than *h/2* in the beginning and will increase to a maximum value of *h/2* as the DCB is loaded. As the crack propagates, the bridging zone will also move with the crack, but the crack face opening displacement will remain as h at C with $2w(0)=d_s(0)=h$. The terms V_z and M_x, respectively, are the bending moment and transverse shear force on the beam cross section.

One should note that the bridging length c is still an unknown and we need an equation to determine c. It can be determined from the fact that the strain energy release rate at the actual crack tip should be equal to the Mode I fracture toughness at the instant of crack propagation. The energy release rate can be determined from the equation derived by Sankar [12] as

$$G = \frac{\left(M(c)\right)^2}{bEI} = \frac{EI}{b}\left(\frac{d^2w(c)}{dx^2}\right)^2 \tag{7}$$

Thus the condition for determining c is

$$\frac{EI}{b}\left(\frac{d^2w(c)}{dx^2}\right)^2 = G_{Ic} \tag{8}$$

Non-dimensional model

Before we solve the above equations we will non-dimensionalize the equations and BCs appropriately. Normalizing the length dimensions by h and forces by Eh^2, the governing equation and the BCs take the following form:

$$\frac{d^4\tilde{w}}{d\tilde{x}^4} - 2\tilde{p}_m\tilde{w} = -\tilde{p}_{\mathrm{m}} \tag{9}$$

$$\begin{aligned}
&\tilde{w}(0) = \tilde{w}_0, \left(0 < w_0 \le 1/2\right) \\
&\frac{d^2\tilde{w}}{d\tilde{x}^2}(0) = \tilde{a}_p \frac{d^3\tilde{w}}{d\tilde{x}^3}(0) \\
&\tilde{w}(\tilde{c}) = 0 \\
&\frac{d\tilde{w}}{d\tilde{x}}(\tilde{c}) = 0
\end{aligned} \tag{10}$$

where $\tilde{x} = \frac{x}{h}, \tilde{a}_p = \frac{a_p}{h}, \tilde{c} = \frac{c}{h}, \tilde{w} = \frac{w}{h}$ and $\tilde{p}_m = \frac{12p_m}{E}$

The equation for determining $\tilde{c}$ (Eq. (8)) takes the form

$$\left(\frac{d^2\tilde{w}(\tilde{c})}{d\tilde{x}^2}\right)^2 = \tilde{G}_{Ic} \tag{11}$$

where the non-dimensional fracture toughness is given by $\tilde{G}_{Ic} = \frac{12G_{Ic}}{Eh}$

The solution for the governing Eq. (9) is

$$\tilde{w}(\tilde{x}) = C_1 \cos\tilde{\lambda}\tilde{x} + C_2 \sin\tilde{\lambda}\tilde{x} + C_3 \cosh\tilde{\lambda}\tilde{x} + C_4 \sinh\tilde{\lambda}\tilde{x} + \frac{1}{2} \tag{12}$$

where $\tilde{\lambda} = \sqrt[4]{2\tilde{p}_m}$

The boundary condition at the point $\tilde{x} = 0$ varies since deflection at this point $\tilde{w}_0$ increases from zero at the beginning of loading to 0.5 when bridging zone is completely developed. Once bridging zone is completely developed this value remains constant at 0.5 with additional increment of $\tilde{a}_p$ since the fully developed bridging zone is preserved and it propagates as the crack tip advances.

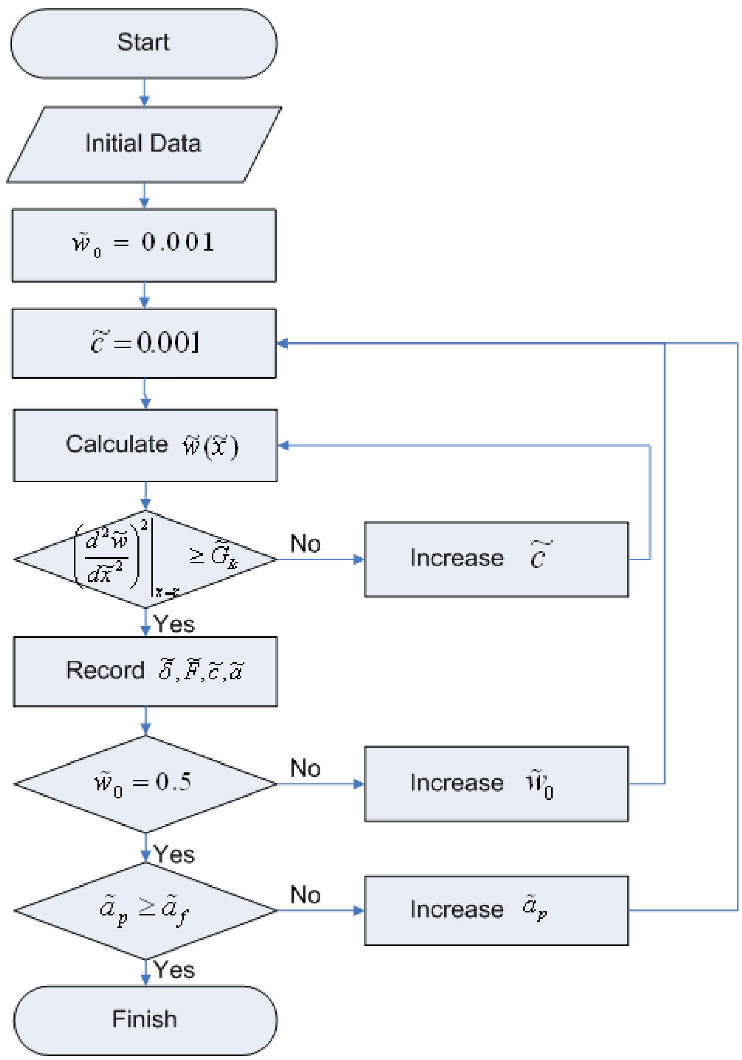

Figure 3: Flowchart of the procedures for solving the non-dimensional governing equation

The procedures to solve the above set of equations are depicted in Fig. 3. The initial data includes the beam properties, characteristics of the z-pins and the fracture toughness G_{Ic}. The deflection at the left end of the bridging zone ($\tilde{x} = 0$) begins to increase as the load is applied. We need to use an iterative procedure as shown as the bridging length ($\tilde{c}$) is not known *a priori*. The strain energy release rate condition at the right end of the bridging zone ($\tilde{x} = \tilde{c}$) as given by Eq. (11) is then used to check for correct value of $\tilde{c}$. After bridging zone is fully developed, bridging length corresponding to every increment of crack length can be determined.

RESULTS FROM THE NON-DIMENSIONAL MODEL

The numerical values used in the examples are given in Table 1. Bridging length and crack length were computed for the case of $\tilde{p}_m = 1.37 \times 10^{-4}$ and $\tilde{G}_{Ic} = 1.4 \times 10^{-5}$.

Table 1: The various dimensions and properties used in the numerical simulation

b	20 mm
h	1.6 mm
E_1	138 Gpa
E_2	11 GPa
ν_{12}	0.34
G_{12}	4.4 GPa
G_{Ic}	258 N/m
a_p	54 - 64 mm

The force F acting on the DCB and opening displacement δ at the end of the beam can be obtained using the following relations:

$$F = -EI \left. \frac{d^3 w}{dx^3} \right|_{x=0} \tag{13}$$

$$\delta = 2\left(w(0) + a_p \left. \frac{dw}{dx} \right|_{x=0} + \frac{1}{3} \left. \frac{d^3 w}{dx^3} \right|_{x=0} a_p^3 \right) \tag{14}$$

The variation of bridging length c and crack length a as a function of DCB deflection δ are shown in Fig. 4. Bridging length begins to increase with loading for a given initial crack length a_0. This bridging length reaches the maximum value when the bridging zone is fully developed. This maximum value starts to decrease slightly as the crack propagates and eventually becomes a constant. This means that bridging length required for fully developed bridging zone is dependent on the initial crack length for transverse loading of the DCB. However, if a pair of moments is applied at the ends of the DCB, the steady state will be preserved and the bridging length will remain a constant as the crack propagates.

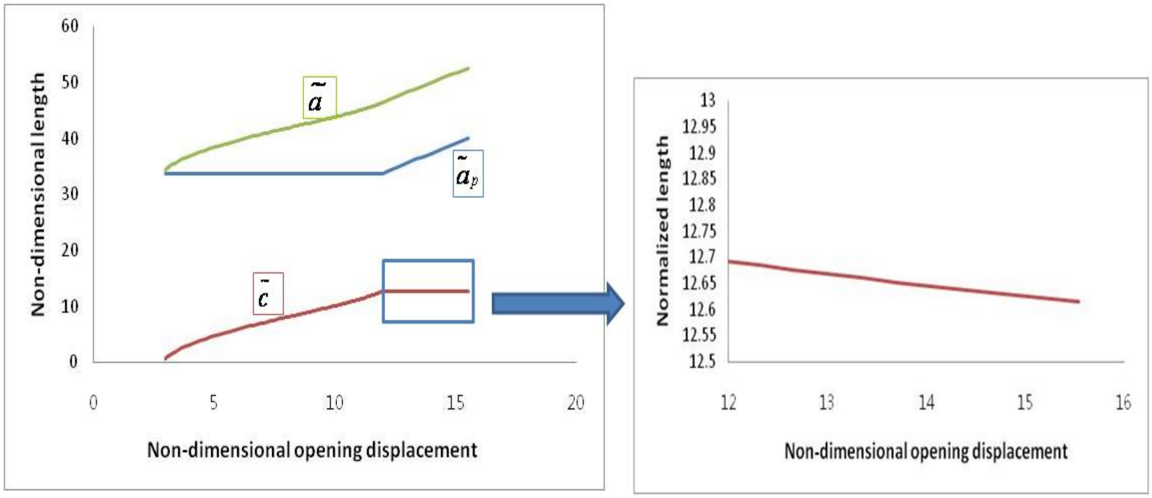

Figure 4. Bridging length and crack length as a function of DCB deflection

Non-dimensional stable bridging length ($\tilde{c}$) can be expressed in terms of non-dimensional fracture toughness $\left(\tilde{G}_c = \dfrac{12G_c}{Eh}\right)$ and non-dimensional frictional force $\left(\tilde{p}_m = \dfrac{12\,p_m}{E}\right)$. Plot of stable bridging length for various $\tilde{G}_c$ and $\tilde{p}_m$ is shown in Fig. 5. According to this result, the value of stable bridging length decreases with increasing $\tilde{G}_c$ for the given $\tilde{p}_m$. Likewise $\tilde{c}$ reduces as $\tilde{p}_m$ increases for a given $\tilde{G}_c$.

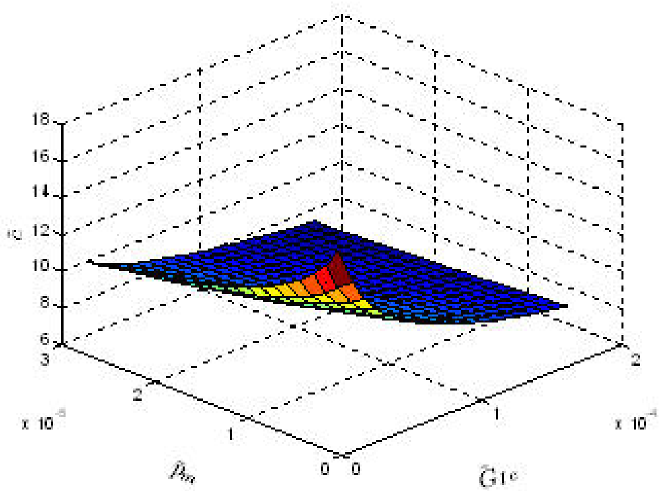

Figure 5. Non-dimensional bridging length

Apparent fracture toughness

Non-dimensional apparent fracture toughness ($\tilde{G}_{Ic-app}$) can be computed as

$$\tilde{G}_{Ic-app} = \left(\frac{d^2\tilde{w}}{d\tilde{x}^2}\right)^2\Bigg|_{\tilde{x}=0} = \tilde{\kappa}(0) \tag{15}$$

where $\tilde{\kappa}(0)$ is the non-dimensional curvature at Point C, beginning of the bridging zone. As shown in Fig. 6 $\tilde{G}_{Ic-app}$ varies linearly with increasing $\tilde{G}_{Ic}$ or $\tilde{p}_m$.

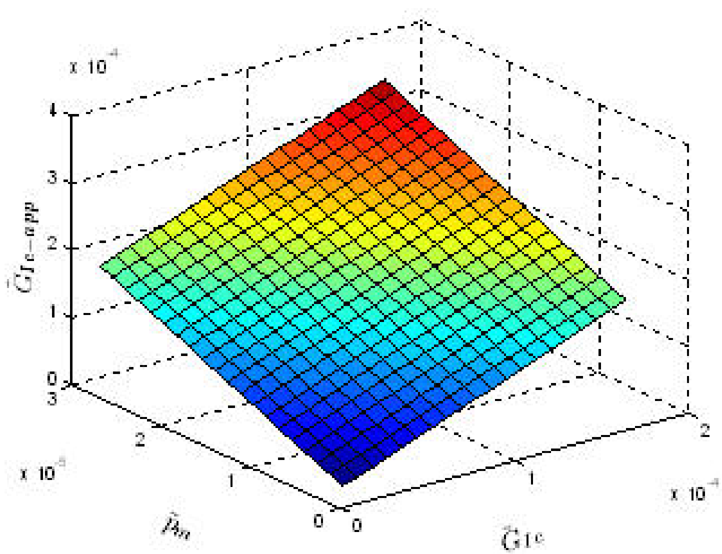

Figure 6. Non-dimensional apparent fracture toughness

The relationship is exactly same as in the equation based on energy balance. As the crack propagates it has to overcome the frictional forces in the z-pins. The amount of extra work done is equal to the area under the load-deflection diagram in Fig. 2. Thus the apparent fracture toughness can be derived as:

$$G_{Ic-app} = G_{Ic} + N\left(\frac{1}{2} f_m h\right) \tag{16}$$

Multiplying throughout by ($12/Eh$) we obtain the above relation in a non-dimensional form as:

$$\tilde{G}_{Ic-app} = \tilde{G}_{Ic} + \frac{1}{2}\tilde{p}_m \tag{17}$$

Maximum frictional force

Even though large frictional force between the z-pin and the surrounding matrix material is desirable for increased fracture toughness, a frictional force beyond a critical value will cause the beam to fail. The maximum normal strain in a beam cross section is given by

$$\varepsilon_{\max} = \frac{h}{2}\left|\frac{d^2 w}{dx^2}\right| \tag{18}$$

Note the strain is already non dimensional and the RHS of the above equation can be written as

$$\varepsilon_{\max} = \left|\frac{d^2 \tilde{w}}{d\tilde{x}^2}\right| = \tilde{\kappa} \tag{19}$$

where $\tilde{\kappa}$ is actually the non-dimensional curvature. The non-dimensional curvature ($\tilde{\kappa}$) has the maximum value at $\tilde{x} = 0$. Using Eq. (15) we obtain

$$\tilde{\kappa} = \sqrt{\tilde{G}_{Ic} + \frac{1}{2}\tilde{p}_m} \tag{20}$$

Let us assume the allowable strain in the composite is given by ε_u

Then

$$\sqrt{\tilde{G}_{Ic} + \frac{1}{2}\tilde{p}_m} < \varepsilon_u \tag{21}$$

From the above equation one can derive

$$\begin{gathered} \tilde{p}_m < 2\left(\varepsilon_u^2 - \tilde{G}_{Ic}\right) \\ \text{or} \\ Nf_m < \frac{E\varepsilon_u^2}{6} - \frac{2G_{Ic}}{h} \end{gathered} \tag{22}$$

The above equation provides an upper limit on the z-pin density which should be taken into consideration in the design of translaminar reinforcements.

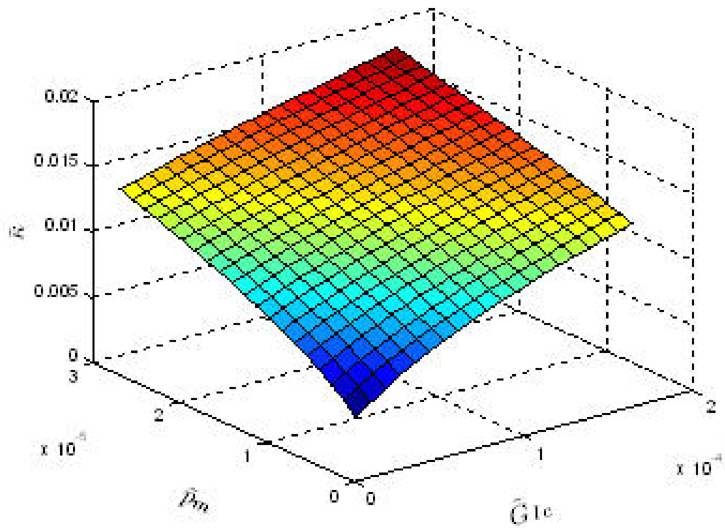

Figure 7. Non-dimensional curvature as a function of maximum friction force and inherent fracture toughness

FE SIMULATION

Three-dimensional FE analysis was used to simulate crack propagation in the DCB specimen with z-pins. Each z-pin was modeled separately by using a discrete element whose behavior is defined by linearly decreasing force as shown in Fig. 2. The configuration of DCB specimen is depicted in Fig.8. The material properties used are given in Tables 1 and 2.

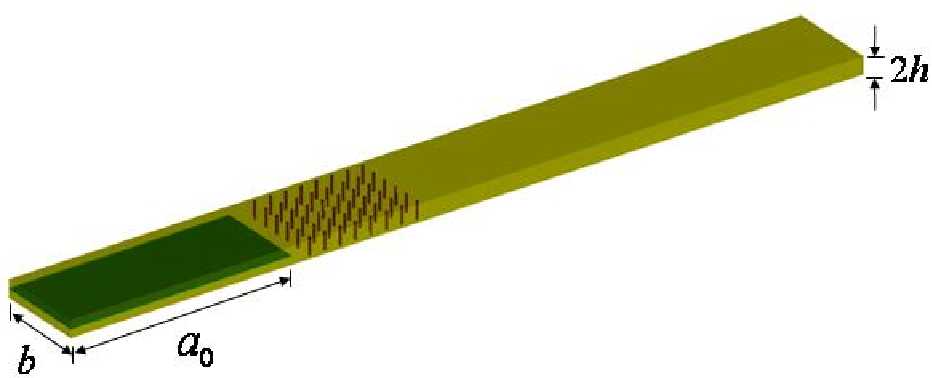

Figure 8. The configuration of DCB specimen [5]

Table 2. Configuration parameters of the DCB

a_0	49 mm
z-pin density	0.5 %
z-pin diameter	0.28 mm
F_m	18.43 N

Delamination propagation in between the z-pins is simulated using cohesive elements. This element is characterized by bi-linear traction separation law [11] given by

$$T = (1 - D)Kd$$
$$D = \begin{cases} 0 & , d < d_0 \\ \dfrac{d_f\left(d - d_0\right)}{d\left(d_f - d_0\right)} & , d_0 < d < d_f \\ 1 & , d_f < d \end{cases} \tag{23}$$

The above parameters were taken as: $K = 10^6$ N/mm, $\sigma_m = 50$ Mpa and $G_{Ic} = 258$ N/m

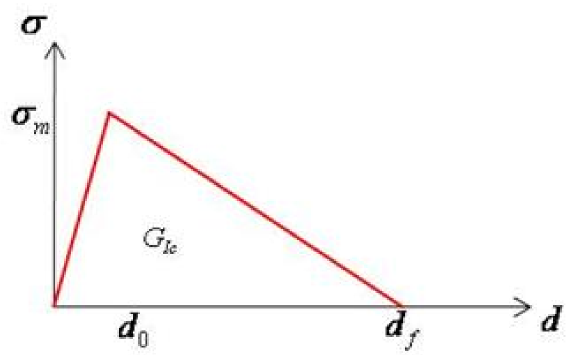

Figure 9. Traction- separation law for the cohesive element

FE analysis was conducted using ABAQUS®. Shell element (S4) was used for DCB and cohesive element (COH3D8) and nonlinear spring element (CONN3D2) were implemented between two ligaments of the DCB as shown in Fig.10.

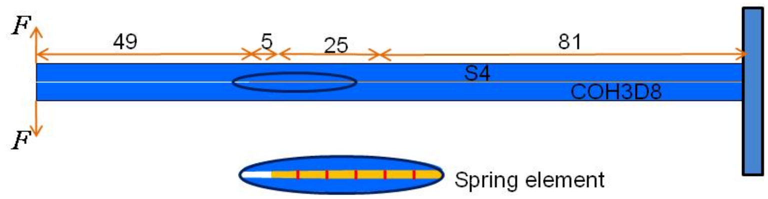

Figure 10. Cohesive and spring elements in the FE model of the DCB

The load-displacement curve from the FE simulation is shown in Fig.11. In the same figure the delamination length *a* is also shown as a function of the opening displacement δ. The agreement between the analytical model and the FE simulations is very good for both load-displacement and delamination length.

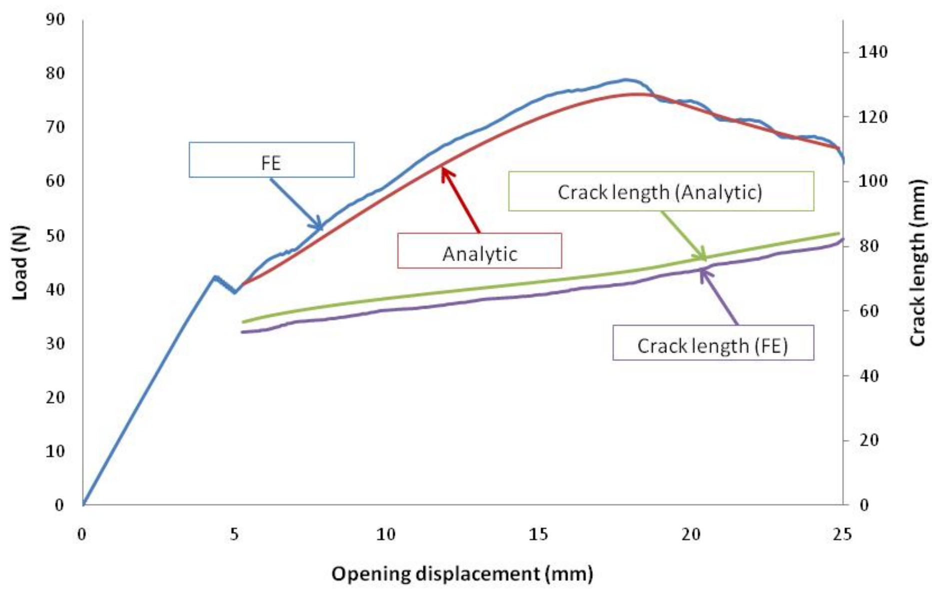

Figure 111. Load-displacement curve and crack length variation as a function of DCB deflection

SUMMARY AND CONCLUSIONS

Mode I delamination propagation in DCB specimens containing z-pins is studied. A simple analytical model has been developed and suitable non-dimensional parameters have been identified. The load-deflection curve of the DCB specimen was calculated using the analytical model. It is seen that the bridging zone, wherein the pins are partially pulled out, develops as the crack propagates, but attains a steady state value. The length of the bridging zone is a function of the inherent Mode I fracture toughness and the frictional force between the z-pins and the surrounding material. An expression was derived for the apparent or effective fracture toughness. Although increase in frictional force as the z-pins increases the fracture toughness, there is an upper limit to this friction as the DCB ligaments would break if the friction is very high. The limiting value of the pin friction is derived.

The efficacy of the analytical model was evaluated by simulation the DCB specimen using finite elements. In the FE model the delaminatiom propagation is simulated by cohesive elements and the z-pins are modeled as discrete non-linear elements. The results for load-deflection curve and the crack bridging zone length agreed quite well with the analytical model.

The non-dimensional model with few parameters will serve as a design tool when translaminar reinforcements such as z-pins are selected for laminated composite structures in order to improve their fracture toughness. The analytical models will also be useful in optimization studies and simulation of large composite structures containing translaminar reinforcements.

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